

Step 4
Normalisation

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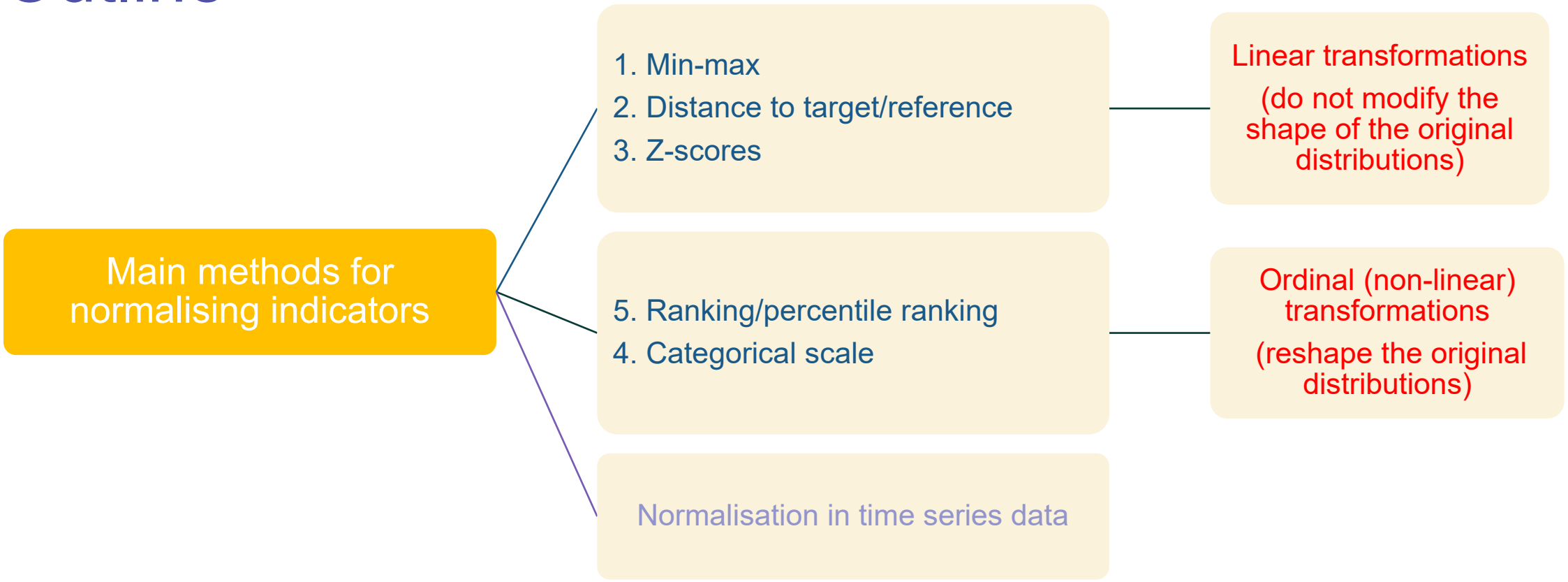
10-Steps to Composite Indicators & Scoreboards



Normalisation

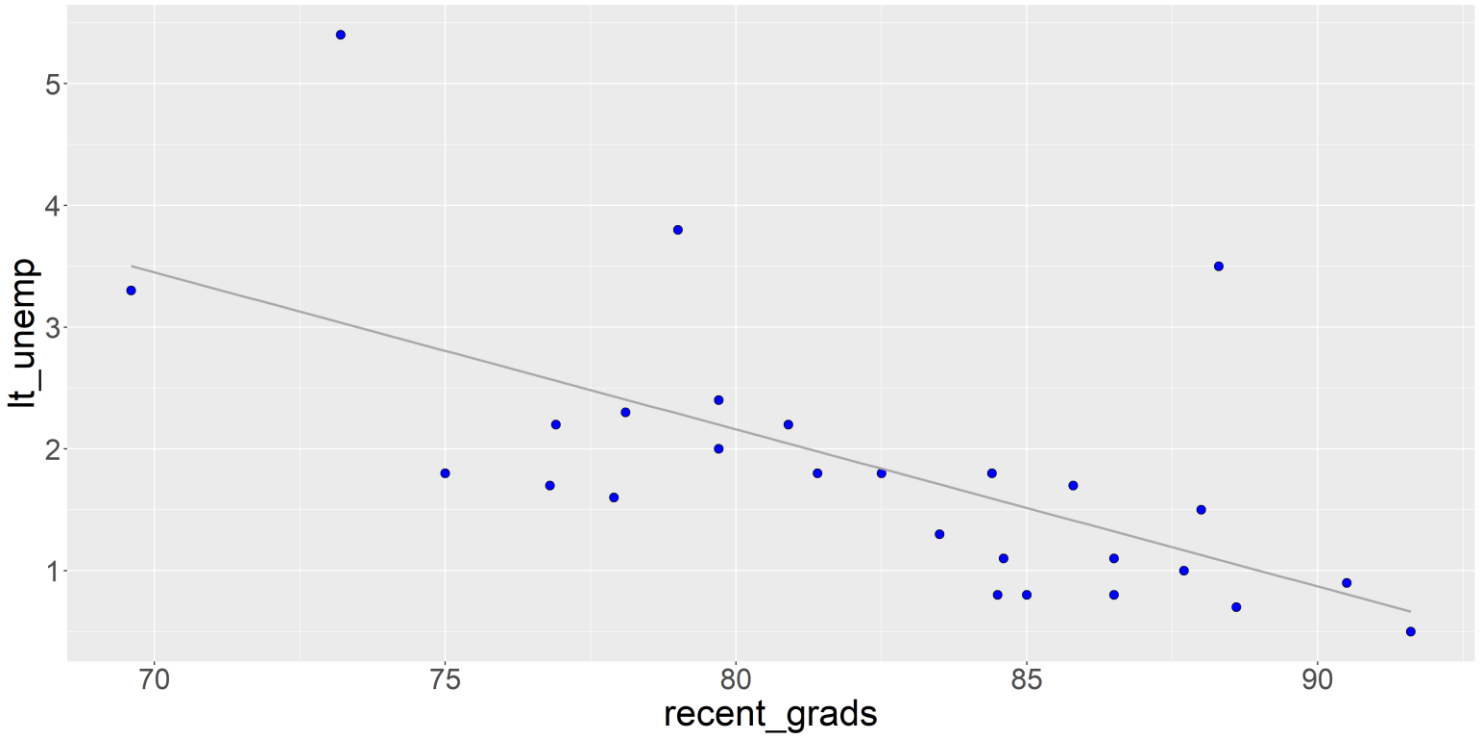
- *What is it?*
  - **Transforms the** (raw) **values of the indicators into a meaningful common numerical scale**
- *Why are we doing it?*
  - To enable **direct comparisons of performance** across indicators
  - To build meaningful **summary performance metrics** when using **additive/multiplicative averages**
- *When shall we apply it?*
  - Whenever we have to adjust for the presence of:
    - **different units of measurement** (e.g. euros vs. percentages)
    - **different distributions and/or variance (variability)**
    - **different "direction"** of the indicators' measurement scales

Outline



N.B.: Different normalisation methods may lead to different rankings in the final composite indicator– the impact of the alternative options can always be explored at the “Robustness & Sensitivity” analysis step

Example: Recent graduates in employment (+) and long-term unemployment rate (-)



r = -0.64

- **Long-term unemployment rate: %** of the economically active population aged 15 to 74 who has been unemployed for 12 months or more.
- **Recent graduates in employment: %** of employed persons aged 20-34 having completed upper secondary or tertiary education 1 to 3 years before the reference year of the survey and who are no longer in education or training
- The indicators have **different direction** (raw values negatively correlated)

Linear transformations

- A linear transformation formula creates a new variable by **adding (or subtracting) a constant and/or multiplying (dividing)** the variable **by another constant**.

  Constants: reference values (e.g. goalposts), descriptive statistics (mean, variance)

- Linear transformation formulas **do not modify the basic shape of the distribution** of the original variable:
  - **skewness** and **kurtosis values** do not change
  - **Values of pairwise** (Pearson) **correlation coefficients** between the indicators are not affected
  - They do not adjust for the presence of **extreme values**

Min-max

$$I_j = \frac{X_j - g_{low}}{g_{up} - g_{low}} = \frac{X_j - \min(X)}{\max(X) - \min(X)} \overset{\text{if sample min.= 0}}{=} \frac{X_j}{\max(X)}$$

For indicators with **positive direction** (reverse direction during normalisation using 1 - *I*)

Effects of normalisation:

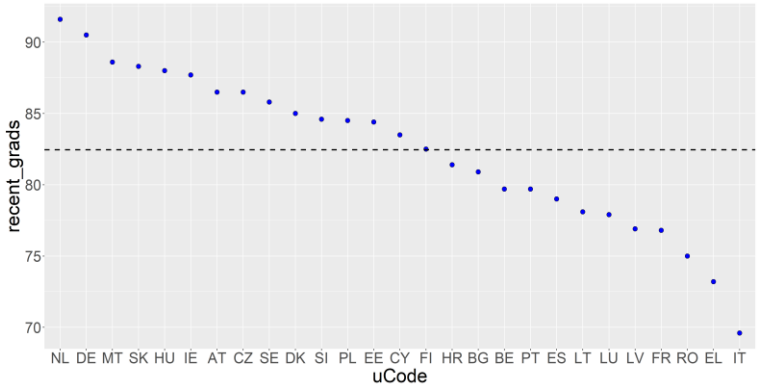
- Unifies scale of measurement
- Unifies range: [0, 1]
- μ, σ^2 not equal
- Extreme values can be identified
- Shape of distribution: no adjustments

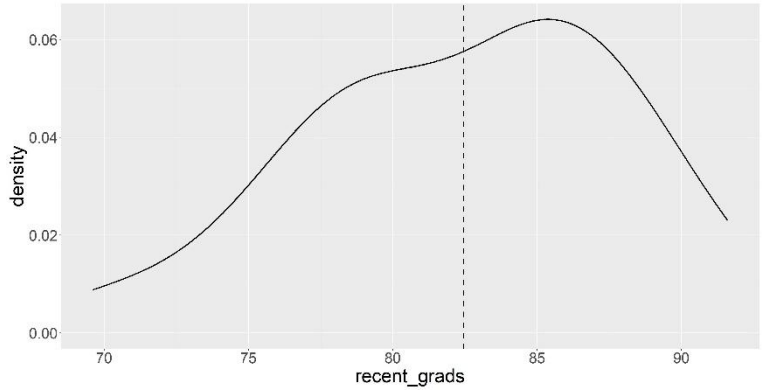
Interpretation of normalised values:

- *relative position (% of distance) between min and max*
- $I_j > 0.5$ → closer to the **max** than the min
- $I_j < 0.5$ → closer to the **min** than the max
- If $\min(X) = 0$, I_j → % of the maximum

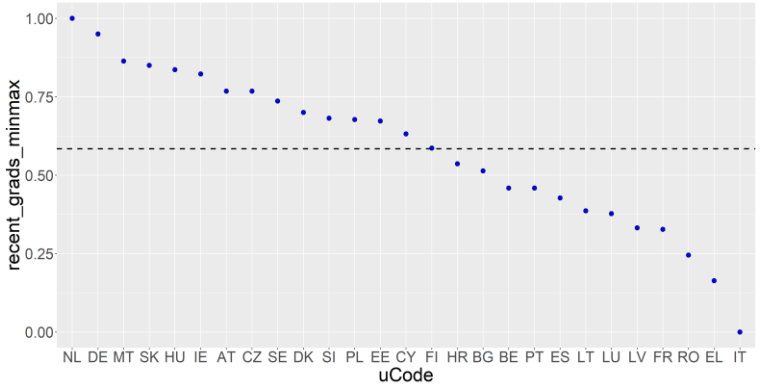
Min-max ((+) direction)

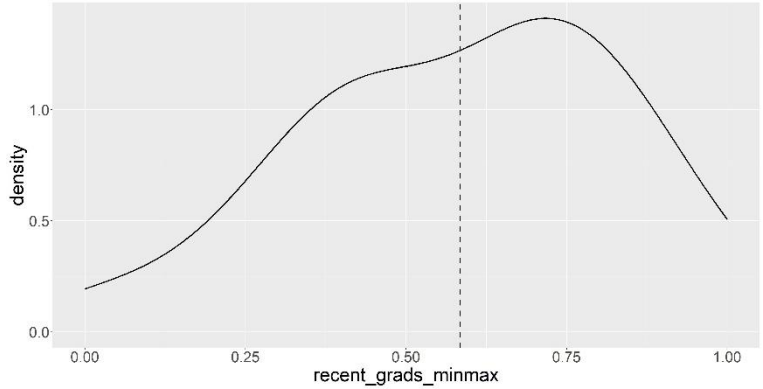
Raw data	mean	var	min	max	range	skew	kurtosis
recent_grads	82.50	30.14	69.60	91.60	22.00	-0.43	-0.33

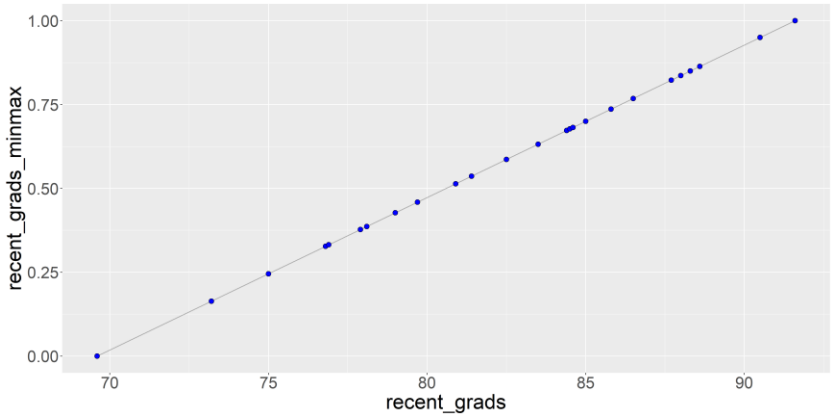




Normalised data	mean	var	min	max	range	skew	kurtosis
recent_grads_**minmax**	0.58	0.06	0.00	1.00	1.00	-0.43	-0.33







Goalposts

if theoretical min.= 0

$$I_j = \frac{X_j - g_{low}}{g_{up} - g_{low}} = \frac{X_j - m_R}{X_R - m_R} \overset{\downarrow}{=} \frac{X_j}{X_R}$$

For indicators with positive direction (reverse direction during normalisation using 1 - I_j)

X_j: raw value country *j*
X_R: raw reference value ("upper" goalpoast – internal or external reference point)
m_R: minimum theoretical value ("lower" goalpost)

Effects of normalisation:

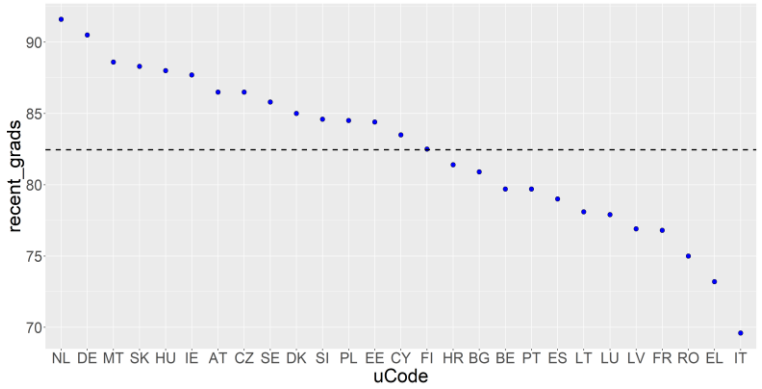
- Unifies scale of measurement
- Range not unified
- μ, σ^2 not equal
- Extreme values can be identified
- Shape of distribution: no adjustments

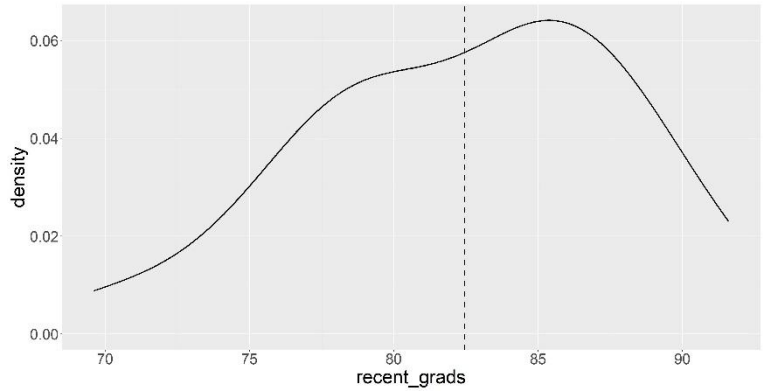
Interpretation of normalised values:

- *"progress" (in %) made from "starting point"* (g_{low}) *towards the target* (g_{up})
- $I_j > 0.5$ → closer to the upper than the lower goalpost or target (and vice versa)
- If $g_{low} = 0$, I_j → % of the goalpost or target

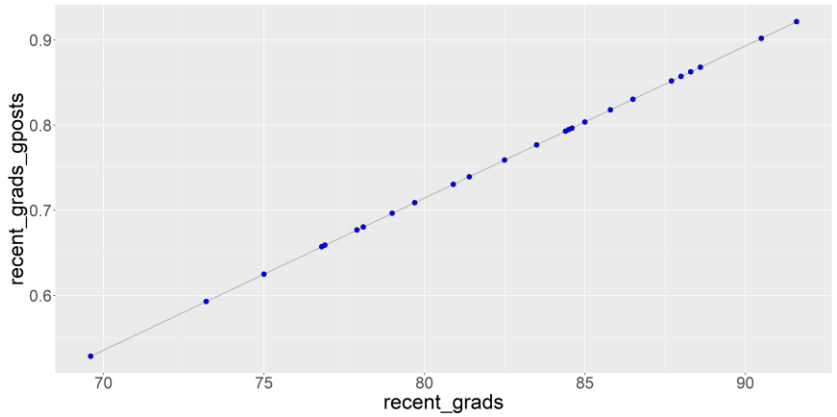
Goalposts ((+) direction)

Raw data	mean	var	min	max	range	skew	kurtosis
recent_grads	82.50	30.14	69.60	91.60	22.00	-0.43	-0.33

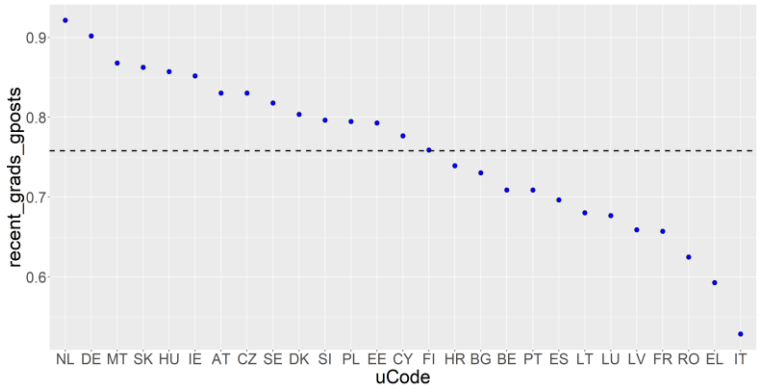


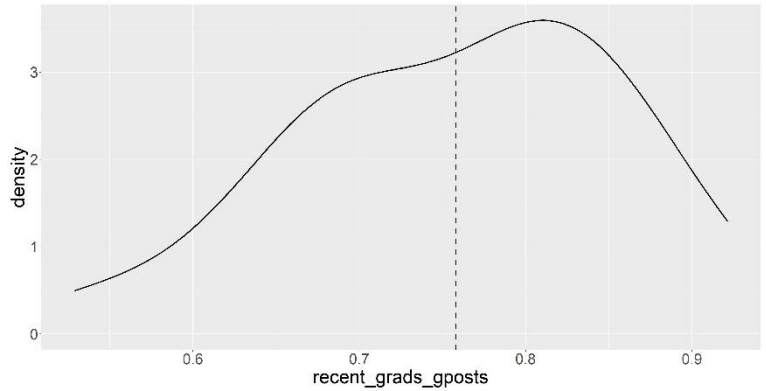


g_{up} = 96 (MT, 2016)
g_{low} = 40 (EL, 2013)

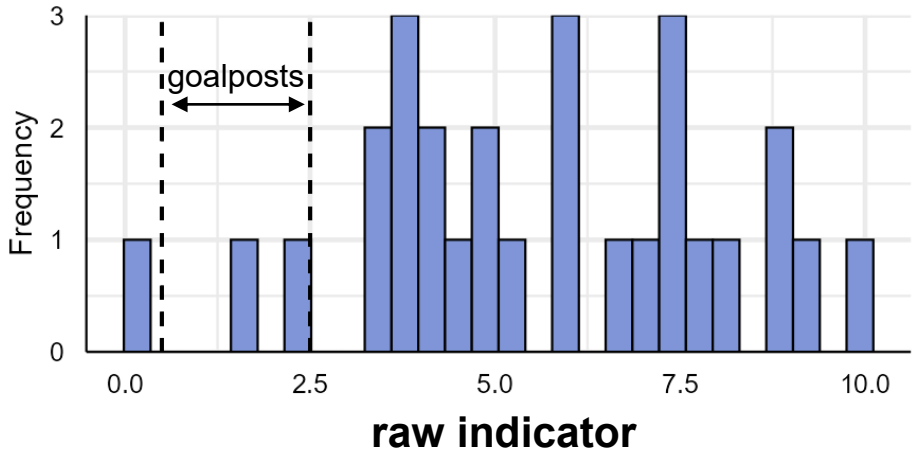


Normalised data	mean	var	min	max	range	skew	kurtosis
recent_grads_**gposts**	0.76	0.01	0.53	0.92	0.39	-0.43	-0.33

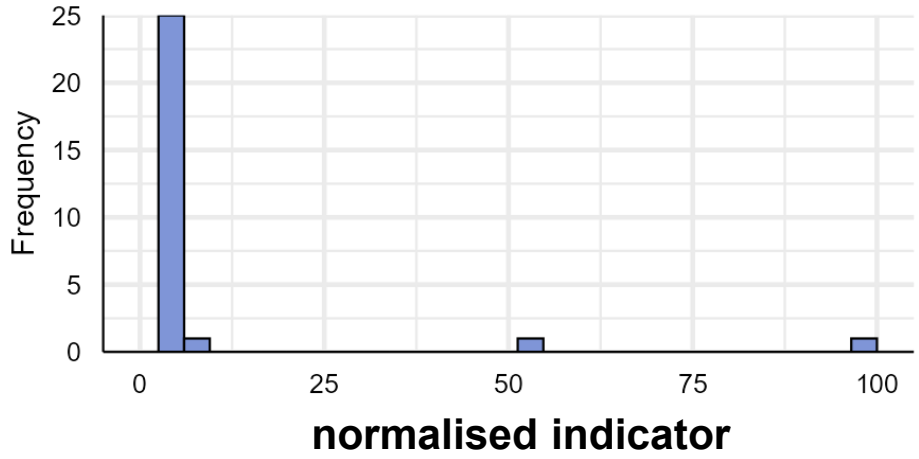




Goalposts selection



- **Conceptual coherence/validity should take priority**
- **Intuitive meaning** that can be **easily communicated**
- **Sources**: *literature, indicator definition, experts, policy docu*
- Things to keep in mind:



1. Your goalposts may have an effect on **outliers**: **Intern**
   BUT:
   - Not anymore a fully linear transformation!
   - Improvements in performance by top (and/or bottom) countries are no longer rewarded
   - Be aware & **<u>investigate</u>** how this affects the distribution of normalised data
2. In **time series** data, goalposts definition requires extra thought (*more on that below*)

Goalposts selection

- Conceptual coherence/validity should take priority
- Intuitive meaning that can be easily communicated
- Sources: *literature, indicator definition, experts, policy documents*
- Things to keep in mind:

1. Your goalposts may have an effect on outliers: Internal goalpost & “capping” the values *BUT:*
    - Not anymore a fully linear transformation!
    - Improvements in performance by top (and/or bottom) countries are no longer rewarded
    - Be aware & **investigate** how this affects the distribution of normalised data
2. **In time series data, goalposts definition requires extra thought (*more on that below*)**

Z-scores

$$Z_j = \frac{X - \bar{X}}{sd(X)}$$

For indicators with positive direction (reverse direction during normalisation multiplying Z_j by *-1*)

Effects of normalisation:

- Unifies scale of measurement
- Range not unified
- Unifies μ = 0 , $\sigma^2 = 1$
- Extreme values can be identified
- Shape of distribution: no adjustments

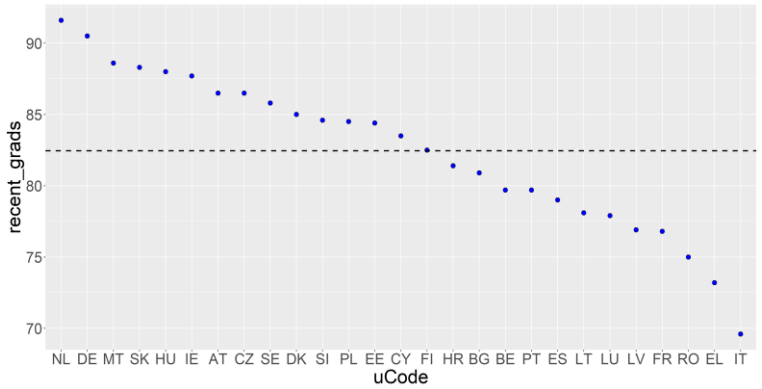
Interpretation of normalised values:

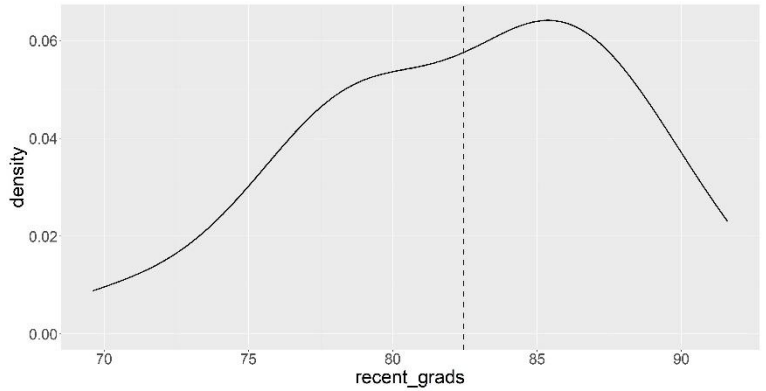
- *number of standard deviations above/below the mean*
- $Z_j = 0$ → raw value equals sample mean
- $Z_j = 1$ → raw value is 1 s.d. > the mean
- $Z_j = -1$ → raw value is 1 s.d. < the mean

N.B: When calculating **geometric averages** of normalised data, the presence of **negative values** in z-score normalised data will result into distorted rankings that do not accurately reflect country performance.

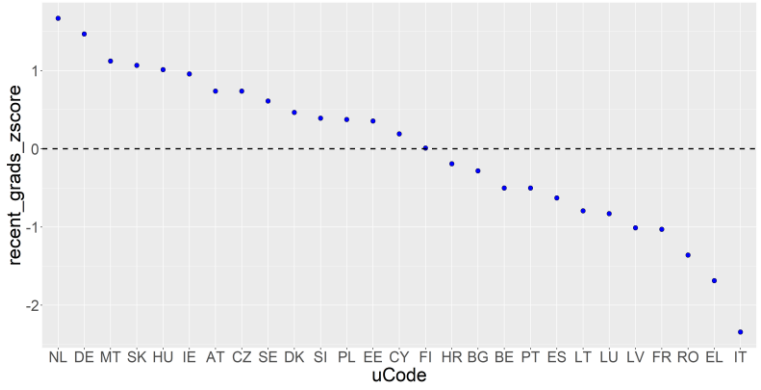
Z-scores ((+) direction)

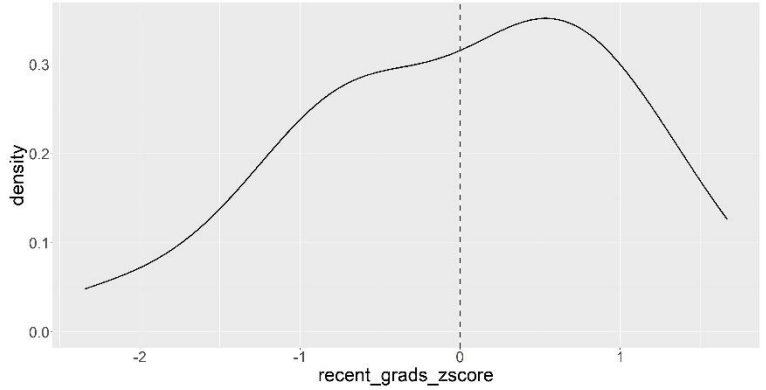
Raw data	mean	var	min	max	range	skew	kurtosis
recent_grads	82.50	30.14	69.60	91.60	22.00	-0.43	-0.33

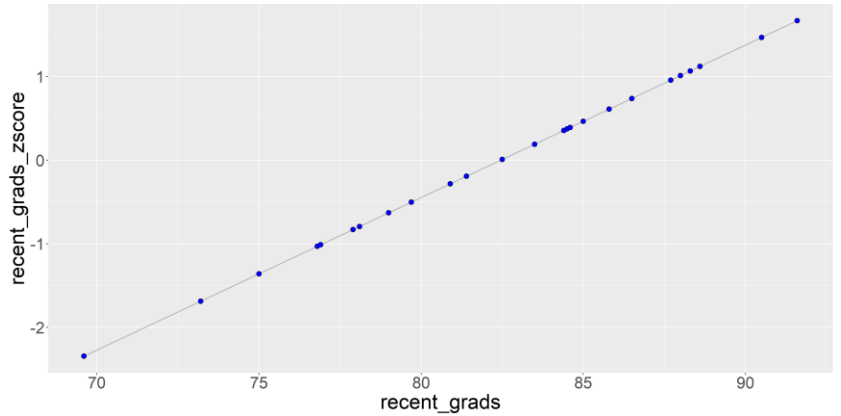




Normalised data	mean	var	min	max	range	skew	kurtosis
recent_grads_**zscore**	0.00	1.00	-2.34	1.67	4.01	-0.43	-0.33







Ordinal (non-linear) transformations

- **Transform the shape of the original distribution** of values.
- **Impact the** (Pearson) **correlation metrics** across indicators in the normalised data set.
- Any **information regarding differences in levels** that might be present in the original indicators **is lost** in their normalised form.
- Normalised values are **not affected by the presence of extreme values** in the original data set
  - **extreme values can no longer be identified** (disappear) **in the transformed data set**.

Ranking

$I_j = rank^d$ (X_j)

where $rank^d$ is the rank in ***descending order*** *of the raw values.* for indicators with positive direction the highest value would receive rank *#1* and the lowest value rank *#N* (**reverse direction during normalisation using *N - rank*^{d} (X_j)**)

Effects of normalisation:

- Unifies scale of measurement
- Unifies range: [1, N]
- Unifies μ, σ^2
- Extreme values cannot be identified
- Reshapes and unifies distributions

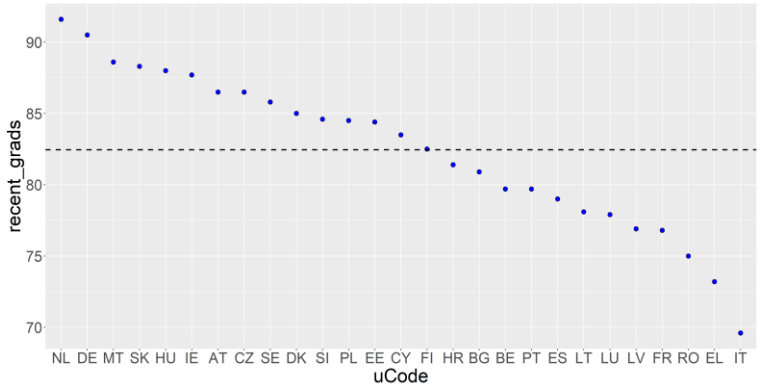
Interpretation of normalised values:

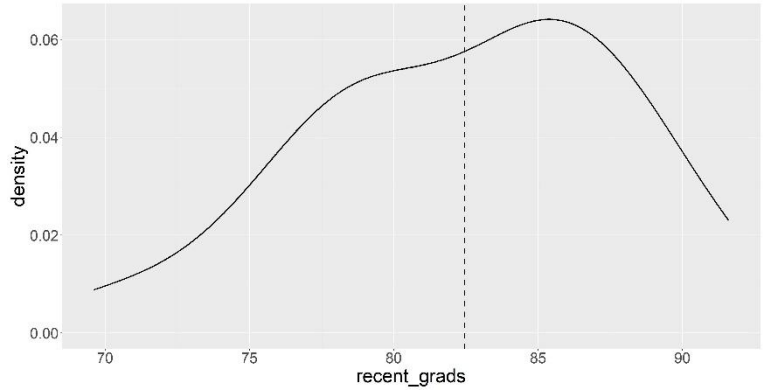
- *performance rankings*
- $I_j = 1$ → top-performer (for that indicator)
- $Z_j = N$ → bottom performer (for that indicator)

N.B: **In the presence of missing values, percentile-ranks** could be considered as an alternative to rankings: each raw value is replaced by the corresponding *p*-th percentile value. This secures a unified range of normalised values (0, 100] in the transformed indicators.

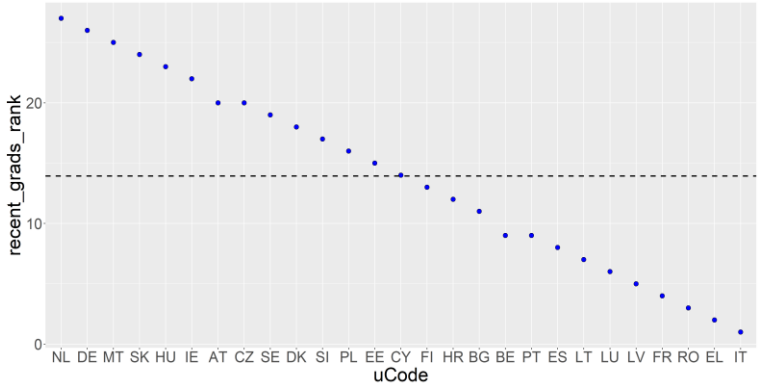
Ranking ((+) direction)

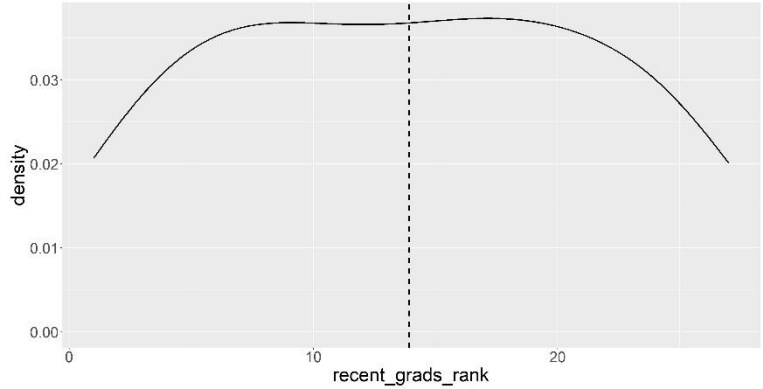
Raw data	mean	var	min	max	range	skew	kurtosis
recent_grads	82.50	30.14	69.60	91.60	22.00	-0.43	-0.33

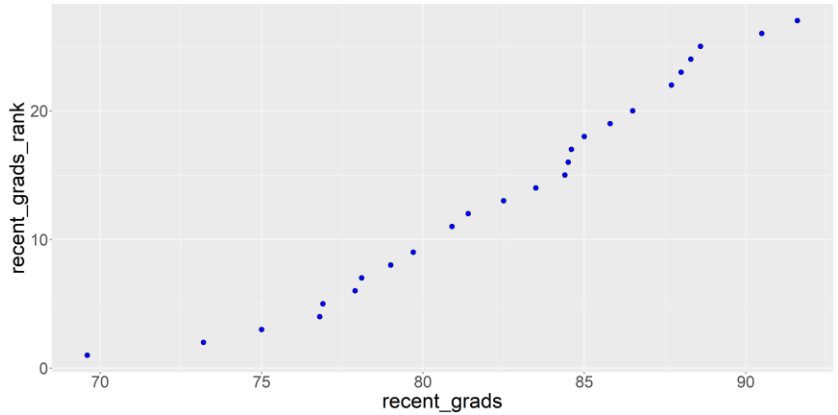




Normalised data	mean	var	min	max	range	skew	kurtosis
recent_grads_**rank**	13.9	62.88	1.00	27.00	26.00	-0.01	-1.20







Categorical scales

$$I_j = \begin{cases} 0 & \text{if } p^0 \leq X_j < p^{10} \\ 20 & \text{if } p^{10} \leq X_j < p^{25} \\ 40 & \text{if } p^{25} \leq X_j < p^{50} \\ 60 & \text{if } p^{50} \leq X_j < p^{75} \\ 80 & \text{if } p^{75} \leq X_j < p^{90} \\ 100 & \text{if } p^{90} \leq X_j \leq p^{100} \end{cases}$$

(**reverse direction during normalisation multiplying *x* by *-1***)

Comparability across two normalized indicators I_i and I_j needs the use of **the same classification system** for them.

Effects of normalisation:

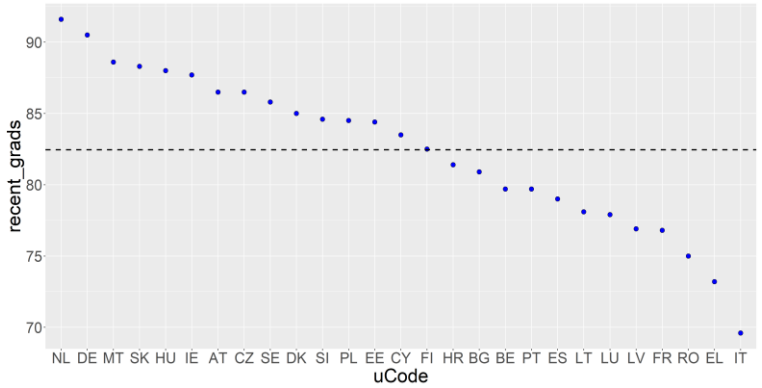
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- Unifies μ, σ^2
- Extreme values cannot be identified
- Reshapes and unifies distributions

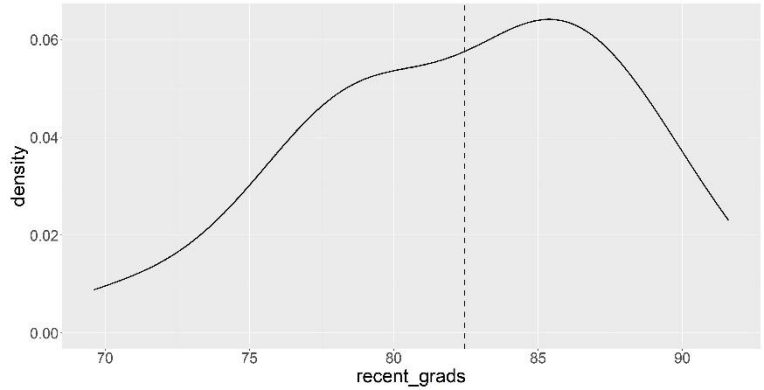
Interpretation of normalised values:

- *Obtained directly from the groups definition*
- e.g. $I_j = 40$ → raw value between 25th and 50th percentile (i.e., below median but above the bottom 25% performers)
- e.g. $I_j = 100$ → raw value among the top 10%

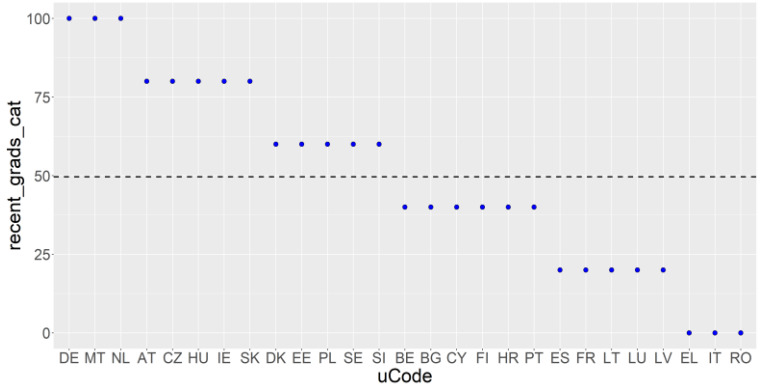
Categorical scales ((+) direction)

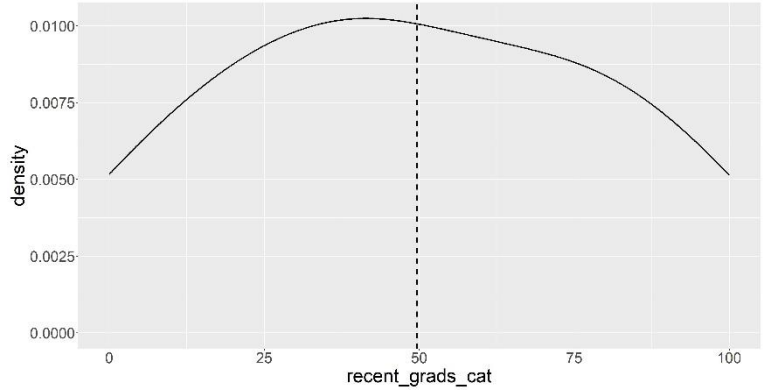
Raw data	mean	var	min	max	range	skew	kurtosis
recent_grads_minmax	82.50	30.14	69.60	91.60	22.00	-0.43	-0.33

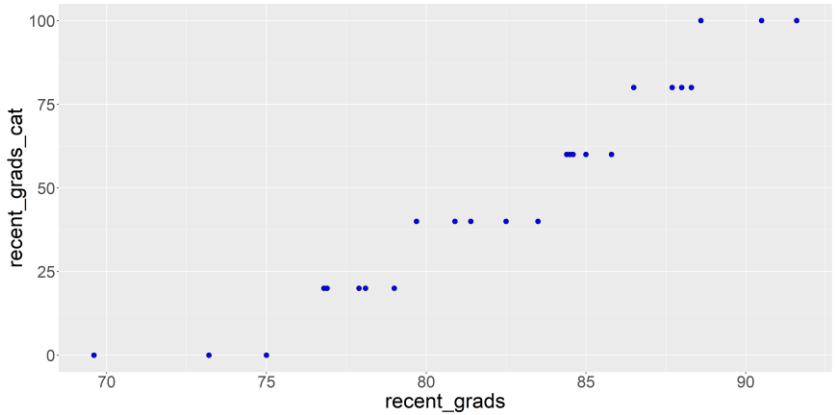




Normalised data	mean	var	min	max	range	skew	kurtosis
recent_grads_**cat**	49.60	967.21	0.00	100.00	100.00	0.03	-1.00





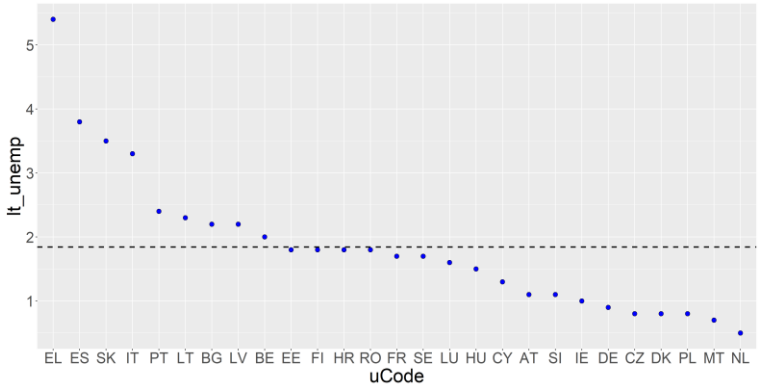


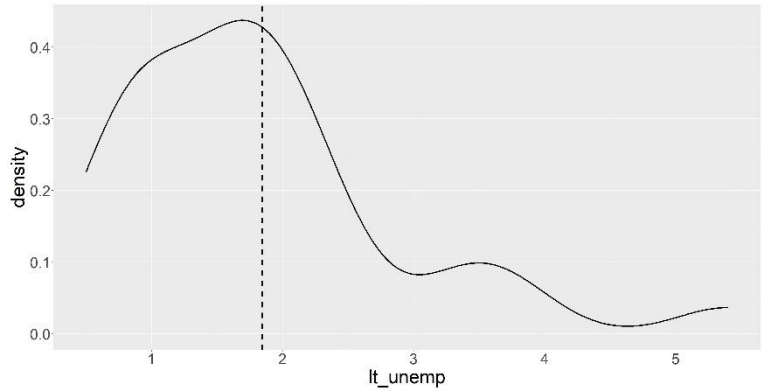
Indicators with negative direction

- "**the-less-the-better**", "negative polarity", "reverse", "non-isotonic" indicators
- Linear transformations:
  - "Reverse" the original distribution of values **without transforming its shape**
  - "Reverse" the **sign** of skewness, kurtosis, and Pearson correlations
- Ordinal (non-linear) normalisation methods
  - "Reverse" the original distribution of values **and transform its shape**
  - "Reverse" the sign **and impact the value of** skewness, kurtosis, and Pearson correlations
- When **all** indicators have negative direction, it may be desired to reflect that also in the composite

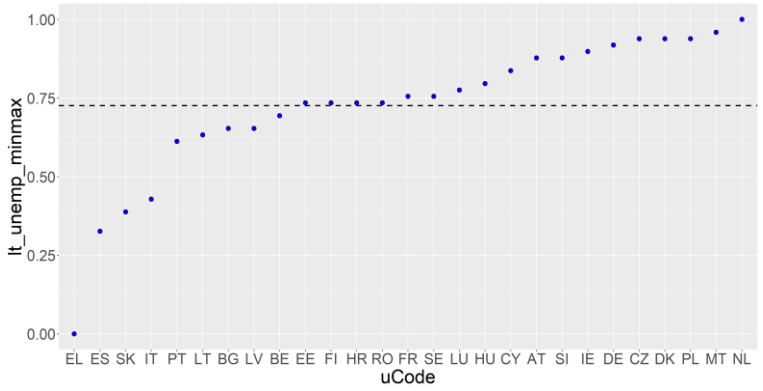
Min-max ((-) direction)

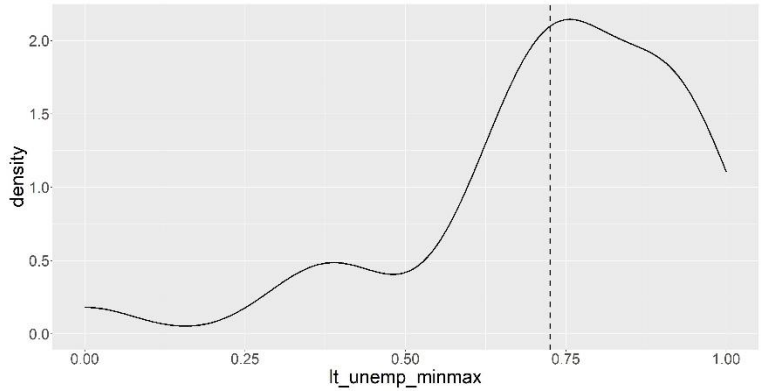
Raw data	mean	var	min	max	range	skew	kurtosis
lt_unemp	1.84	1.23	0.50	5.40	4.90	1.56	3.03

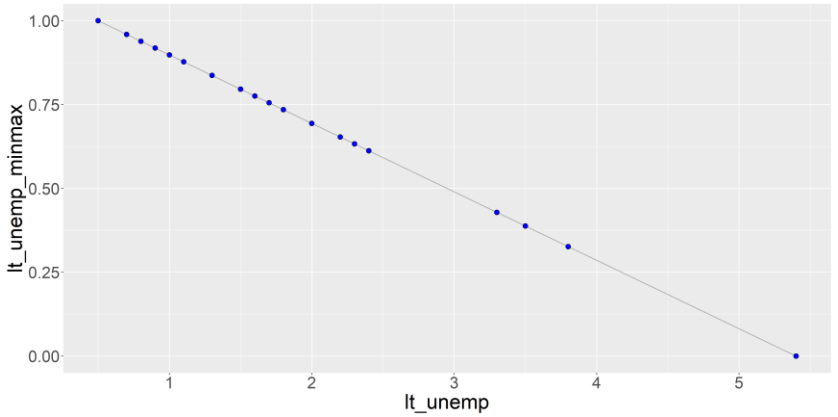




Normalised data	mean	var	min	max	range	Skew	kurtosis
lt_unemp_**minmax**	76.38	24.63	64.80	82.90	18.10	-1.56	-3.03

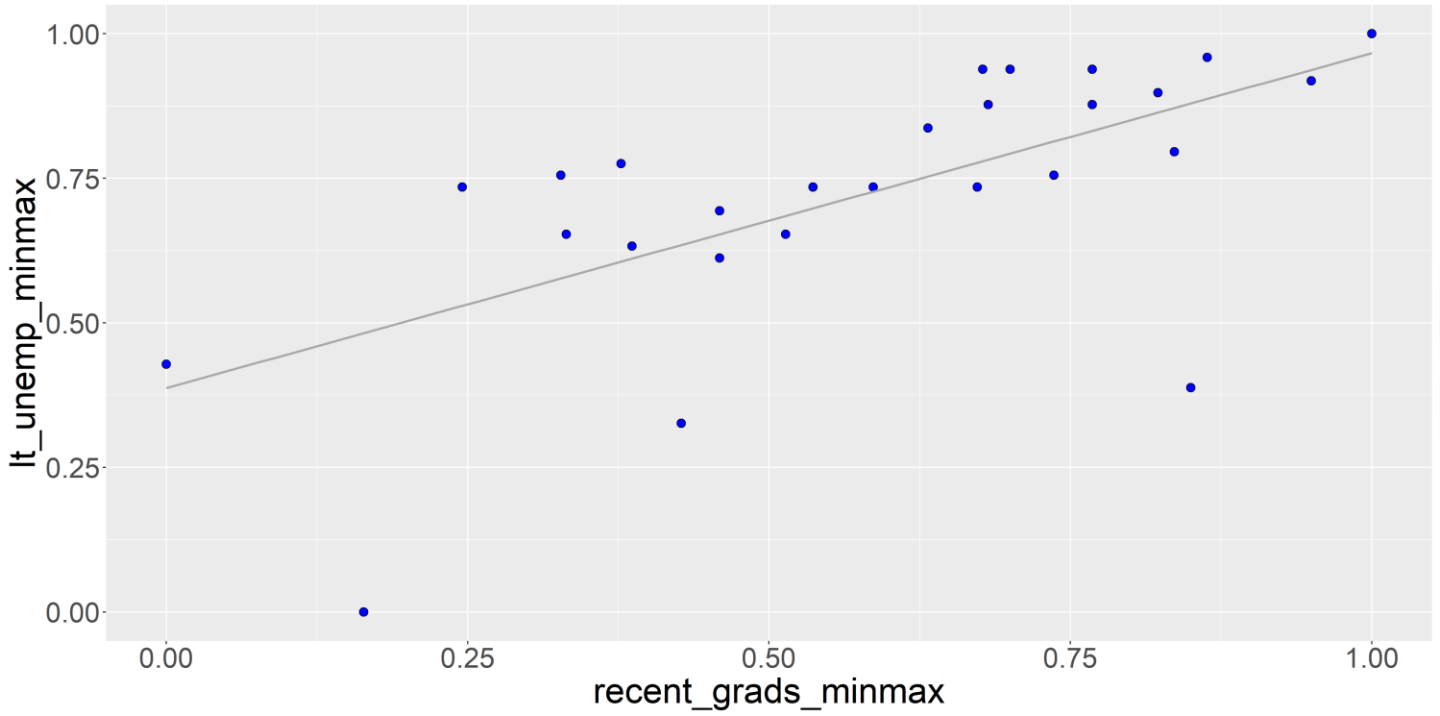






Relation of normalised indicators ((+) and (-) direction)

Normalisation: min-max





r = 0.64

Normalisation of time series data

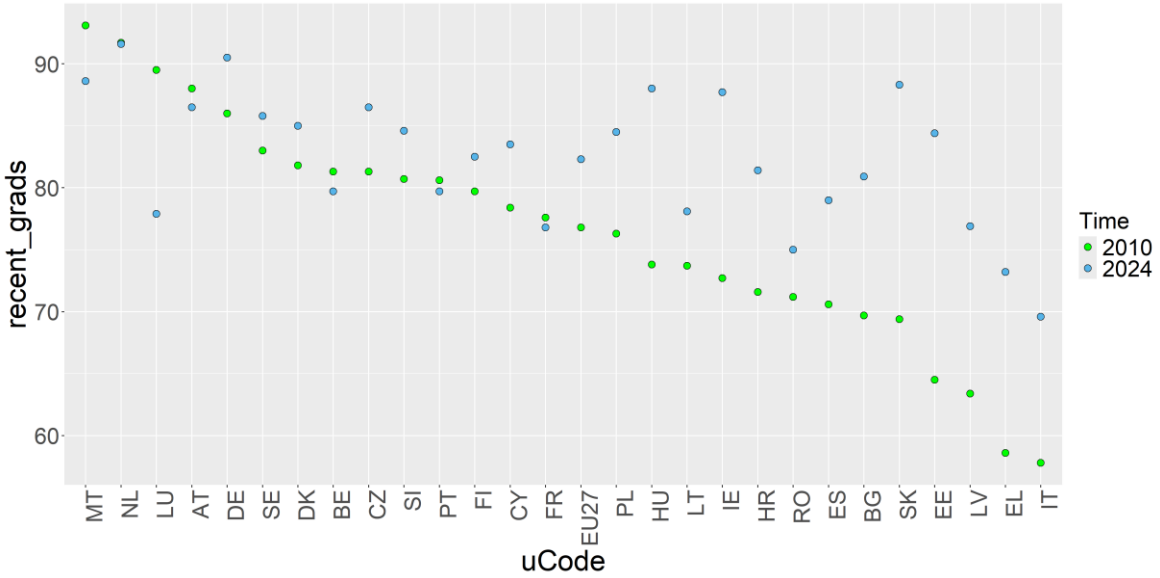
- For comparable data ***across entities and time*** *we need:*
  - **common benchmarks** (*max, min, average, sd, goalposts, base year & country*)
  - **consider all years of data** when calculating *ranks or categorical scales.*
- Data transformations relying on sample estimates (*e.g., min-max*) are *not stable* when data for new years are included. Re-calculation of normalised values is necessary.
- Normalisation of data with respect to a **base year and country** *(e.g., EU27 in 2010)* is not sensitive to the addition of new data (normalised data for existing years do not change)
- Normalised data by goalposts are not affected by new years of data as long as goalposts do not need to change.
- ***Communication of the normalisation method and potential concerns over comparability with previous editions of the composite indicator or scoreboard are of outmost importance***

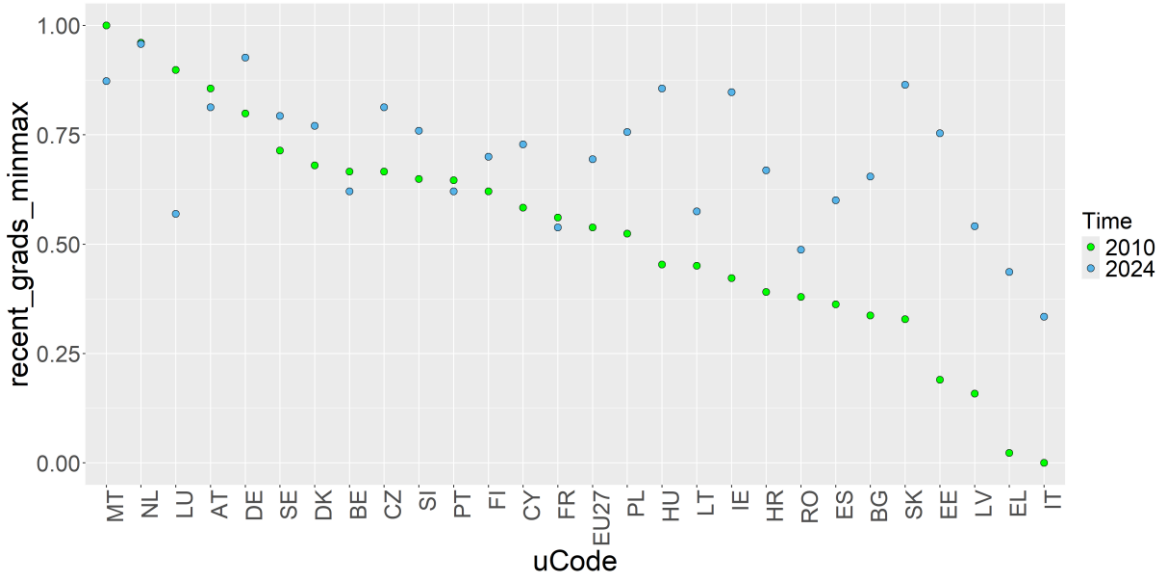
Normalisation of time series data: Min-max

- Variable: recent graduates in employment
- Years: 2010 & 2024

Raw data	mean	var	min	max	range	skew	kurtosis
recent_grads	79.50	64.32	57.80	31.10	35.30	-0.70	0.37

Normalised data	mean	var	min	max	range	skew	kurtosis
recent_grads_**minmax**	0.61	0.05	0.00	1.00	1.00	-0.70	0.37





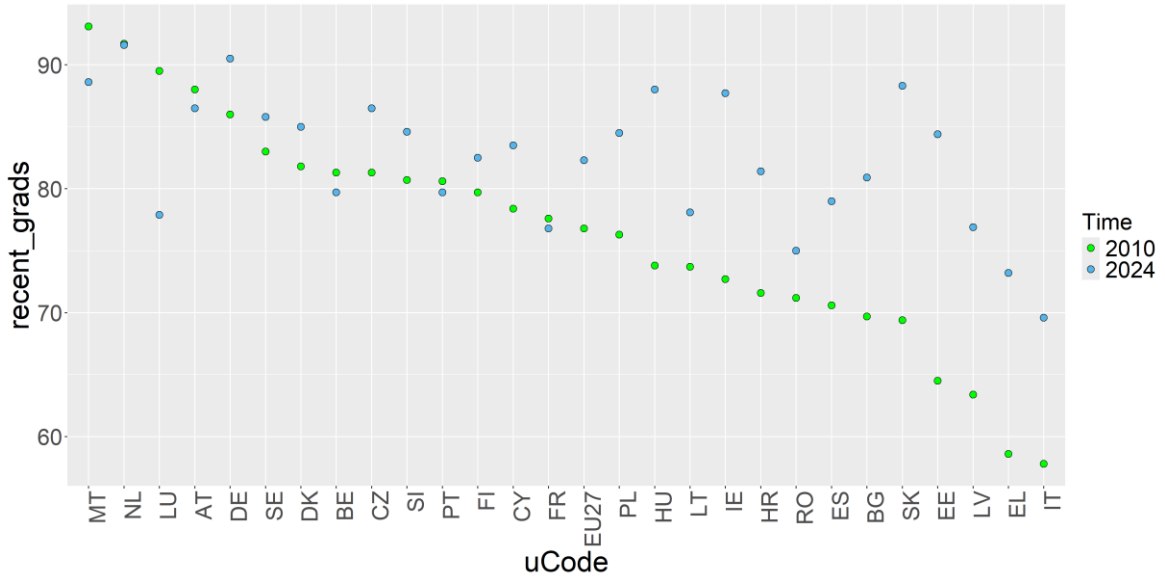
Normalisation of time series data: base year & country

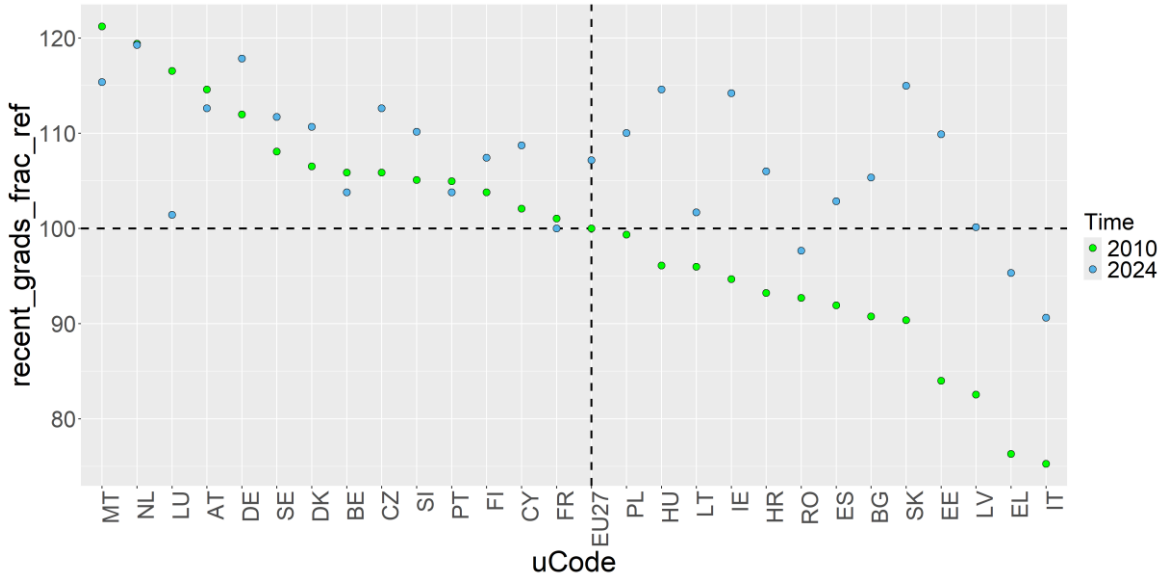
- Variable: recent graduates in employment
- Years: 2010 & 2024

Base year & country: EU27 (2010)
Normalisation: percentage of raw value $I_j = \left(X_j / 76{,}8\right) * 100$

Raw data	mean	var	min	max	range	skew	kurtosis
recent_grads	79.50	64.32	57.80	31.10	35.30	-0.70	0.37

Normalised data	mean	var	min	max	range	skew	kurtosis
recent_grads_**frac_ref**	103.00	108.16	75.30	121.00	45.7	-0.70	0.37





Normalised values interpreted as percentage changes with respect to the EU27-2010 baseline (100)

Take-away messages

- **Linear vs non-linear (ordinal) transformation of data?**
  - *The information contained in **raw quantitative variables** is always welcome*
- **Is there a desired interpretation of the normalised data?**
  - *What is easier for my audience to grasp?*
- **Target values or reference points to compare with?**
  - *e.g.1 EU-level 2030 targets for certain indicators*
  - *e.g.2 EU27 aggregate score for the 27 Member States*
- **Is there a time dimension?**
  - *One-off assessment? Future releases of the indicator (with new years of data) planned?*
- **Assess the impact of different normalisation methods via uncertainty and sensitivity analysis**

Thank you

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https://knowledge4policy.ec.europa.eu/composite-indicators_en

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